International Journal of Theoretical Physics, Vol. 38, No. 2, 1999

Coupling of Tachyons to Electromagnetism

C. G. Bollini,[1] L. E. Oxman,[2] and M. C. Rocca[1]

Received August 19, 1998

We consider a fourth-order equation implying a bradyonic and a tachyonic mode of propagation for a scalar field. The electromagnetic field is introduced via the gauge covariant derivative. We show that the interacting fourth-order equation is equivalent to a second-order Klein–Gordon equation also minimally coupled, with the tachyon living in closed loops connected only to photon lines. The equivalence shows that the fourth-order theory is renormalizable and unitary.

1. INTRODUCTION

For a discussion on the possible existence of tachyons see Feinberg [1].

Clasically a superluminal charged particle will emit Čerenkov radiation, losing energy and momentum to the electromagnetic field [2]. Quantum mechanically we should consider a field obeying a Klein–Gordon equation with the wrong sign of the mass term [3, 4]. However, a close examination of this equation [5, 6] leads to the conclusion that such a field can not exist asymptotically as a free wave. Its propagation is not of the Feynman type, but rather of the Wheeler type, i.e., half-advanced and half-retarded [7]. Consequently a tachyon cannot be observed as a free particle. For this reason it seems that the question of the possible existence of an electrically charged superluminal particle loses much of its meaning, as the very definition of charge involves the scattering of a free particle by a potential.

Nevertheless, a tachyon field can exist, together with other fields, as a carrier of interactions, or as a virtual particle between interaction vertices. The tachyon may act as a companion field for normal particles.

[1] Departamento de Fisica, Faculdad de Ciencias Exactas, Universidad Nacional de La Plata, C.C. 67 (1900) La Plata, Argentina.
[2] Departamento de Fisica Teorica, Universidade Federal do Rio de Janeiro, C.P. 68528 Rio de Janeiro, RJ, 21945-970 Brazil.

0020-7748/99/0200-077 7$16.00/0

A simple example of the coexistence of bradyons and tachyons is provided by the equation [8]

$$(\Box^2 - m^4)\varphi = (\Box - m^2)(\Box + m^2)\varphi = 0 \tag{1}$$

The field φ has two modes of propagation. One mode leads to normal (bradyonic type) Klein–Gordon particles. The other leads to tachyons (wrong sign of the mass term). The equivalence of a higher order equation to a set of different excitations was studied in ref. 9.

The Green function of eq. (1) can be defined by the Fourier transform of (see also ref. 8)

$$(\Box^2 - m^4)\tilde{G} = i\delta \tag{2}$$

i.e.,

$$G = \frac{i}{p^4 - m^4} \tag{3}$$

while for a normal Klein–Gordon field we have

$$(\Box - m^2)\tilde{K} = i\delta \tag{4}$$

$$K = \frac{-i}{p^2 + m^2} \tag{5}$$

Note that

$$\frac{i}{p^4 - m^4} = \frac{1}{2m^2}\left(\frac{-i}{p^2 + m^2} + \frac{i}{p^2 - m^2}\right) \tag{6}$$

where the bradyon and tachyon modes can explicitly be seen. Note the relative normalization factor $(2m^2)^{-1}$.

We can tackle our main problem by considering the coupling of φ [obeying Eq. (1)] with the electromagnetic field. In this way we expect to see the mutual influence of electromagnetism and the field φ, in its two modes of propagation, thus indirectly answering the question of the possible existence of a charged tachyon.

The interaction will be introduced via the gauge-invariant "minimal coupling" procedure. We replace each derivative in Eq. (1) by

$$\partial_\mu \to \partial_\mu - ieA_\mu \tag{7}$$

For the D'Alembertian, this well-known procedure gives

$$\begin{aligned} \Box \to \Box' &\equiv (\partial^\mu - ieA^\mu)(\partial_\mu - ieA_\mu) \\ &= \Box - 2ieA\cdot\partial - e^2A^2 \end{aligned} \tag{8}$$

where we have chosen the Lorentz gauge $\partial_\mu A^\mu = 0$.

The right-hand side of (8) implies that for a normal scalar particle we have a first order interaction:

$$I_1 = 2e\varepsilon \cdot p \tag{9}$$

where ε_μ is the polarization vector of the photon and p_μ is the four-momentum of the particle.

The second-order term is

$$I_2 = -2e^2\varepsilon_1 \cdot \varepsilon_2 \tag{10}$$

2. INTERACTION VERTICES

It is easy to deduce the result of the minimal coupling replacement [Eqs. (7) and (8)] for the iterated D'Alembertian:

$$\begin{aligned}
\Box^2 \Rightarrow \Box'^2 &= (\Box - 2ieA \cdot \partial - e^2A^2) \\
&\quad \cdot (\Box - 2ieA \cdot \partial - e^2A^2) \\
\Box'^2 &= \Box^2 - 2ie(\Box A \cdot \partial + A \cdot \partial\Box) \\
&\quad - e^2(\Box A^2 + A^2\Box) - 4e^2A \cdot \partial A \cdot \partial \\
&\quad + 2ie^3(A \cdot \partial A^2 + A^2A \cdot \partial) + e^4A^4
\end{aligned} \tag{11}$$

This procedure gives rise to four interaction terms:

First order:

$$J_1 = -2e\varepsilon \cdot p_1(p_1^2 + p_2^2), \qquad p_2 = p_1 + k \tag{12}$$

Second order:

$$J_2 = 2e^2\varepsilon_1 \cdot \varepsilon_2(p_1^2 + p_2^2) + 4e^2(\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2 + \varepsilon_1 \cdot p_2\varepsilon_2 \cdot p_1) \tag{13}$$

Third order:

$$J_3 = -4e^3(p_1 + p_2) \cdot (\varepsilon_1\varepsilon_2 \cdot \varepsilon_3 + \varepsilon_2\varepsilon_1 \cdot \varepsilon_3 + \varepsilon_3\varepsilon_1 \cdot \varepsilon_2) \tag{14}$$

Fourth order:

$$J_4 = 8e^4(\varepsilon_1 \cdot \varepsilon_2\varepsilon_3 \cdot \varepsilon_4 + \varepsilon_1 \cdot \varepsilon_3\varepsilon_2 \cdot \varepsilon_4 + \varepsilon_1 \cdot \varepsilon_4\varepsilon_2 \cdot \varepsilon_3) \tag{15}$$

Note that the interaction seems to be of the nonrenormalizable type. Compare, for example, J_1 [Eq. (12)] with I_1 [Eq. (9)]. However, the propagator

(3) has an extra power of two in the denominator, so that by power counting, the theory turns out to be renormalizable. Furthermore, we are going to show that it is equivalent to the second order theory for a charged scalar particle determined by Eqs. (5) and (8)–(10).

To see the equivalence in a clear way, we are going to examine a couple of examples. It will be evident that the theory given by the fourth-order equation (1) with the gauge invariant electromagnetic coupling [Eq. (11)] shares the same properties with the above-mentioned second-order theory.

For the construction of the matrix elements we must remember that any vertex needs an i from the perturbative expansion, besides the factor I [Eqs. (9) and (10)] or J [Eqs. (12)–(15)]. Also an external scalar line implies a propagator, either K [Eq. (5)] or G [Eq. (3)].

3. COMPTON EFFECT

The lowest order Feynman diagrams are shown in Fig. 1.

For the second-order Klein–Gordon theory we use the propagator K [Eq. (5)] and I_1, I_2 [Eqs. (9) and (10)]. We obtain

$$M^{(2)} = (i2e\varepsilon_1 \cdot p_1)\frac{-i}{p^2 + m^2}(i2e\varepsilon_2 \cdot p_2) \cdot (i2e\varepsilon_2 \cdot p_1)\frac{-i}{q^2 + m^2}(i2e\varepsilon_1 \cdot p_2) - i2e^2\varepsilon_1 \cdot \varepsilon_2 \tag{16}$$

For the fourth-order theory, the propagator is G [Eq. (3)] and the vertex factors are J_1 [Eq. (12)] and J_2 [Eq. (13)]:

$$\begin{aligned} M^{(4)} = {} & [-i2e\varepsilon_1 \cdot p_1\,(p_1^2 + p^2)]\frac{i}{p^1 - m^4}[-i2e\varepsilon_2 \cdot p_2(p^2 + p_2^2)] \\ & + [-i2e\varepsilon_2 \cdot p_1\,(p_1^2 + q^2)]\frac{i}{q^4 - m^4}[-i2e\varepsilon_1 \cdot p_2(q^2 + m^2)] \\ & + i2e^2\varepsilon_1 \cdot \varepsilon_2\,(p_1^2 + p_2^2) + 4ie^2\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2 \\ & + 4ie^2\varepsilon_2 \cdot p_1\varepsilon_1 \cdot p_2 \end{aligned} \tag{17}$$

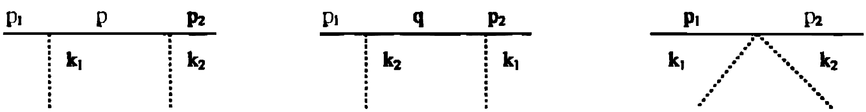

Fig. 1. Compton scattering.

In Eq. (17)) we use the fact that the initial and final states (of φ) correspond to a bradyon, i.e., $p_1^2 = p_2^2 = -m^2$:

$$M^{(4)} = -i4e^2\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2 \frac{p^2 - m^2}{p^2 + m^2} - i4e^2\varepsilon_2 \cdot p_1\varepsilon_1 \cdot p_2 \frac{q^2 - m^2}{q^2 + m^2} + 4ie^2\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2 + 4ie^2\varepsilon_2 \cdot p_1\varepsilon_1 \cdot p_2 - 4m^2ie^2\varepsilon_1 \cdot \varepsilon_2 \tag{18}$$

A comparison of Eqs. (16)) and (18) shows that

$$M^{(4)} = 2m^2M^{(2)} \tag{19}$$

This equality means that for the lowest order Compton effect of the field φ ($M^{(4)}$) we can ignore the tachyon mode of propagation and proceed as if only the bradyon mode were excited ($M^{(2)}$).

It is easy to see that virtual photons do not spoil the equivalence we have found. Take, for example, the production of a photon in the scattering of two charged particles (see Fig. 2). For the vertices corresponding to the virtual photon we should take

$$I'_1 = 2ep_\mu, \qquad I'_2 = -2e^2\varepsilon_\mu$$
$$J'_1 = -2ep^1_\mu(p_1^2 + p_2^2)$$
$$J'_2 = 2e^2\varepsilon_\mu(p_1^2 + p_2^2) + 4e^2p_{1\mu}\varepsilon \cdot p_2 + 4e^2\,p_{2\mu}\varepsilon \cdot p_1 \tag{20}$$

With (20) and following the steps of the evaluation of (16) and (17), we will be able to prove that $M'^{(4)} = 2m^2M'^{(2)}$.

In the introduction we said that the tachyon cannot propagate asymptotically as a free wave. Its propagator is equivalent, on the real axis, to Cauchy's principal value Green function, so that its value on the free particle pole is exactly zero. We are now going to prove not only that the tachyon mode

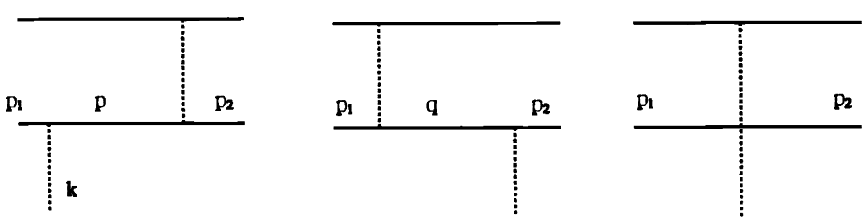

Fig. 2. Virtual photons.

cannot propagate freely, but also that it cannot be produced by the Compton effect on the bradyon mode of φ.

For the proof we take again the amplitude $M^{(4)}$ [Eq. (17)] and put now $p_1^2 = -m^2$ (a bradyon) and $p_2^2 = +m^2$ (a tachyon). The matrix element for this tachyon production is then [cf. Eq. (17)]

$$M = [-i2e\varepsilon_1 \cdot p_1(p^2 - m^2)] \frac{i}{p^4 - m^4} [-i2e\varepsilon_2 \cdot p_2 (p^2 + m^2)]$$
$$+ [-i2e\varepsilon_2 \cdot p_1 (q^2 - m^2)] \frac{i}{q^4 - m^4} [-i2e\varepsilon_1 \cdot p_2(q_2 + m^2)]$$
$$+ i2e^2\varepsilon_1 \cdot \varepsilon_2(-m^2 + m^2) + 4ie^2(\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2 + \varepsilon_2 \cdot p_1\varepsilon_1 \cdot p_2) \equiv 0 \tag{21}$$

and the probability amplitude for the production of a tachyon mode by the Compton effect, turns out to be zero.

4. DOUBLE PHOTON SCATTERING

To see clearly the mechanism for the proof of the equivalence [Eq. (19)], we will consider now a third-order process. The relevant Feynman diagrams are shown in Fig. 3. The C diagram is only valid for the fourth-order theory. For the first diagram we use G [Eq. (3)] and J_1 [Eq. (12)]. We have

$$A_1 = [-2ie\varepsilon_1 \cdot p_1(p_1^2 + p^2)] \frac{i}{p^4 - m^4} [-2ie\varepsilon_2 \cdot p (p^2 + r'^2)]$$
$$\cdot \frac{i}{r'^4 - m^4} [-2ie\varepsilon_3 \cdot p_2(r'^2 + p_2^2)] \tag{22}$$

With $p_1^2 = p_2^2 = -m^2$, we get

$$A_1 = -8ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p_2\varepsilon_3 \cdot p_2 \frac{(p^2 + r'^2)}{(p^2 + m^2)(r'^2 + m^2)}$$

All A diagrams can be obtained mutatis mutandis from A_1:

$$A_2 = -8ie^3\varepsilon_1 \cdot p_1\varepsilon_3 \cdot p\varepsilon_2 \cdot p_2 \frac{(p^2 + q'^2)}{(p^2 + m^2)(q'^2 + m^2)}$$

$$A_3 = -8ie^3\varepsilon_2 \cdot p_1\varepsilon_1 \cdot q\varepsilon_3 \cdot p_2 \frac{(q^2 + r'^2)}{(q^2 + m^2)(r'^2 + m^2)}$$

$$A_4 = -8ie^3\varepsilon_2 \cdot p_1\varepsilon_3 \cdot q\varepsilon_1 \cdot p_2 \frac{(q^2 + p'^2)}{(q^2 + m^2)(p'^2 + m^2)}$$

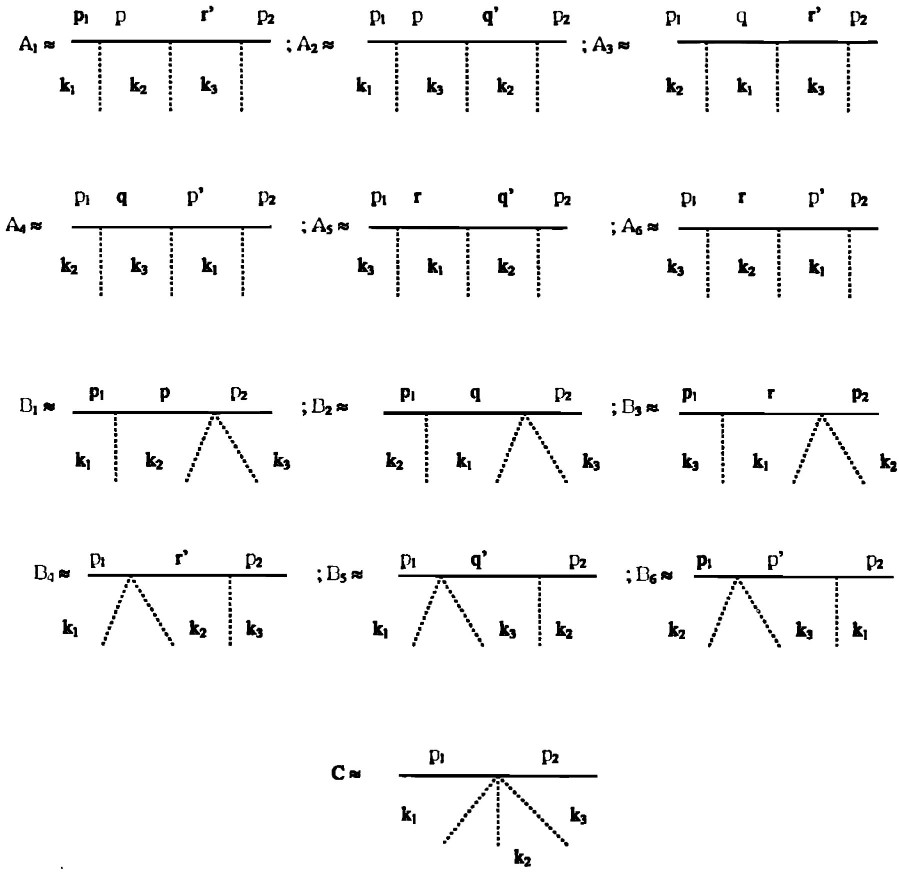

Fig. 3. Double photon scattering.

$$A_5 = -8ie^3\varepsilon_3 \cdot p_1\varepsilon_1 \cdot r\varepsilon_2 \cdot p_2 \frac{(r^2 + q'^2)}{(r^2 + m^2)(q'^2 + m^2)}$$

$$A_6 = -8ie^3\varepsilon_3 \cdot p_1\varepsilon_2 \cdot r\varepsilon_1 \cdot p_2 \frac{(r^2 + p'^2)}{(r^2 + m^2)(p'^2 + m^2)} \tag{23}$$

For the B diagrams we need also J_2 [Eq. (13)]:

$$B_1 = [-2ie\varepsilon_1 \cdot p_1 (p_1^2 + p^2)] \frac{i}{p^4 - m^4} [2ie^2\varepsilon_2 \cdot \varepsilon_3(p_1^2 + p^2)$$
$$+ 4ie^2(\varepsilon_2 \cdot p\varepsilon_3 \cdot p_2 + \varepsilon_3 \cdot p\varepsilon_2 \cdot p_2)]$$

$$B_1 = 4ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot \varepsilon_3 \frac{p^2 - m^2}{p^2 + m^2} + 8ie^3\varepsilon_1 \cdot p_1 \cdot (\varepsilon_2 \cdot p\varepsilon_3 \cdot p_2 + \varepsilon_3 \cdot p\varepsilon_2 \cdot p_2)\frac{1}{p^2 + m^2} \tag{24}$$

With appropiate changes we have

$$\begin{aligned}
B_2 &= 4ie^3\varepsilon_2 \cdot p_1\varepsilon_1 \cdot \varepsilon_3 \frac{q^2 - m^2}{q^2 + m^2} + 8ie^3\varepsilon_2 \cdot p_1 \\
&\quad \cdot (\varepsilon_1 \cdot q\varepsilon_3 \cdot p_2 + \varepsilon_3 \cdot q\varepsilon_1 \cdot p_2)\frac{1}{q^2 + m^2} \\
B_3 &= 4ie^3\varepsilon_3 \cdot p_1\varepsilon_1 \cdot \varepsilon_2 \frac{r^2 - m^2}{r^2 + m^2} + 8ie^3\varepsilon_3 \cdot p_1 \\
&\quad \cdot (\varepsilon_1 \cdot r\varepsilon_2 \cdot p_2 + \varepsilon_2 \cdot r\varepsilon_1 \cdot p_2)\frac{1}{r^2 + m^2} \\
B_4 &= 4ie^3\varepsilon_3 \cdot p_2\varepsilon_1 \cdot \varepsilon_2 \frac{r'^2 - m^2}{r'^2 + m^2} + 8ie^3\varepsilon_3 \cdot p_2 \\
&\quad \cdot (\varepsilon_1 \cdot p_1\varepsilon_2 \cdot r' + \varepsilon_2 \cdot p_1\varepsilon_1 \cdot r')\frac{1}{r'^2 + m^2} \\
B_5 &= 4ie^3\varepsilon_2 \cdot p_2\varepsilon_1 \cdot \varepsilon_3 \frac{q'^2 - m^2}{q'^2 + m^2} + 8ie^3\varepsilon_2 \cdot p_2 \\
&\quad \cdot (\varepsilon_1 \cdot p_1\varepsilon_3 \cdot q' + \varepsilon_3 \cdot p_1\varepsilon_1 \cdot q')\frac{1}{q'^2 + m^2} \\
B_6 &= 4ie^3\varepsilon_1 \cdot p_2\varepsilon_2 \cdot \varepsilon_3 \frac{p'^2 - m^2}{p'^2 + m^2} + 8ie^3\varepsilon_1 \cdot p_2 \\
&\quad \cdot (\varepsilon_2 \cdot p_1\varepsilon_3 \cdot p' + \varepsilon_3 \cdot p_1\varepsilon_2 \cdot p')\frac{1}{p'^2 + m^2}
\end{aligned} \tag{25}$$

For diagram C we use Eq. (14):

$$C = -4ie^3\,(p_1 + p_2) \cdot (\varepsilon_1\varepsilon_2 \cdot \varepsilon_3 + \varepsilon_2\varepsilon_1 \cdot \varepsilon_3 + \varepsilon_3\varepsilon_1 \cdot \varepsilon_2) \tag{26}$$

We first group together all terms in which each polarization vector ε is contracted with some four-momentum vector. We take A_1 plus parts of B_1 and B_4 (with similar $\varepsilon \cdot p$ dependence).

$$A_1' = -8ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p\varepsilon_3 \cdot p_2\left[\frac{p^2 + r'^2}{(p^2 + m^2)(r'^2 + m^2)} - \frac{1}{p^2 + m^2} - \frac{1}{r'^2 + m^2}\right]$$

$$A_1' = 2m^2 8ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot p\varepsilon_3 \cdot p_2 \frac{1}{(p^2 + m^2)(r'^2 + m^2)}$$

Analogously,

$$A_2' = 2m^2 8ie^3\varepsilon_1 \cdot p_1\varepsilon_3 \cdot p\varepsilon_2 \cdot p_2 \frac{1}{(p^2 + m^2)(q'^2 + m^2)}$$

$$A_3' = 2m^2 8ie^3\varepsilon_2 \cdot p_1\varepsilon_1 \cdot q\varepsilon_3 \cdot p_2 \frac{1}{(q^2 + m^2)(r'^2 + m^2)}$$

$$A_4' = 2m^2 8ie^3\varepsilon_2 \cdot p_1\varepsilon_3 \cdot q\varepsilon_1 \cdot p_2 \frac{1}{(q^2 + m^2)(p'^2 + m^2)}$$

$$A_5' = 2m^2 8ie^3\varepsilon_3 \cdot p_1\varepsilon_1 \cdot r\varepsilon_2 \cdot p_2 \frac{1}{(r^2 + m^2)(q'^2 + m^2)}$$

$$A_6' = 2m^2 8ie^3\varepsilon_3 \cdot p_1\varepsilon_2 \cdot r\varepsilon_1 \cdot p_2 \frac{1}{(r^2 + m^2)(p'^2 + m^2)} \tag{27}$$

Finally, let us take the first term from the B-matrix elements, Eqs. (25), together with similar terms from C [Eq. (26)]:

$$B_1' = 4ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot \varepsilon_3 \left(\frac{p^2 - m^2}{p^2 + m^2} - 1\right)$$

$$B_1' = -2m^2 4ie^3\varepsilon_1 \cdot p_1\varepsilon_2 \cdot \varepsilon_3 \frac{1}{p^2 + m^2}$$

$$B_2' = -2m^2 4ie^3\varepsilon_2 \cdot p_1\varepsilon_1 \cdot \varepsilon_3 \frac{1}{q^2 + m^2}$$

$$B_3' = -2m^2 4ie^3\varepsilon_3 \cdot p_1\varepsilon_1 \cdot \varepsilon_2 \frac{1}{r^2 + m^2}$$

$$B_4' = -2m^2 4ie^3\varepsilon_3 \cdot p_2\varepsilon_1 \cdot \varepsilon_2 \frac{1}{r'^2 + m^2}$$

$$B_5' = -2m^2 4ie^3\varepsilon_2 \cdot p_2\varepsilon_1 \cdot \varepsilon_3 \frac{1}{q'^2 + m^2}$$

$$B'_6 = -2m^2 4ie^3 \varepsilon_1 \cdot p_2 \varepsilon_2 \cdot \varepsilon_3 \frac{1}{p'^2 + m^2} \tag{28}$$

We have proved that

$$\sum_{i=1}^{6} A_i + \sum_{i=1}^{6} B_i + C = \sum_{i=1}^{6} A'_i + \sum_{i=1}^{6} B'_i \tag{29}$$

It is easy to check that A'_i and B'_i are ($2m^2$ times) the matrix elements corresponding to diagrams A and B for the Klein–Gordon case, i.e., with the propagator K and the interaction terms I_1 and I_2. So, the equivalence holds also in third order.

Likewise, we can prove that the probability amplitude for the production of a tachyon mode is identically zero in third order of the perturbation expansion.

5. GENERAL PROOF

It is possible to show, order by order, that the equality $M^{(4)} = 2m^2 M^{(2)}$ always holds. However, it is preferable to have a general proof. To that aim we will use functional methods.

The Lagrangian corresponding to the fields φ and A_μ interacting in a gauge-invariant way is

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\varphi}(\Box'^2 - m^4)\varphi \tag{30}$$

where $\Box'$ is defined by Eq. (8). Our generating functional is

$$\mathfrak{L}(\mathcal{T}, \bar{\mathcal{T}}, \mathcal{K}) = \int [\mathfrak{D}\varphi][\mathfrak{D}\bar{\varphi}]\,[\mathfrak{D}A] \exp i \int dx(\mathcal{L} + \zeta(\partial \cdot A)^2 + \mathcal{T}\varphi + \bar{\mathcal{T}}\bar{\varphi} + \mathcal{K} \cdot A) \tag{31}$$

The term in ζ assures the Lorentz gauge for the electromagnetic field.

The exponential in (31) depends quadratically on the scalar field. We can then use the general Gaussian formula [10]

$$\int [\mathfrak{D}\varphi][\mathfrak{D}\bar{\varphi}] \exp i \int dx(\bar{\varphi}\mathbb{O}\varphi + \mathcal{T}\varphi + \bar{\mathcal{T}}\bar{\varphi}) = \mathcal{N}'(Det\,\mathbb{O})^{-2} \exp -i \int dx \bar{\mathcal{T}}\mathbb{O}^{-1}\mathcal{T} \tag{32}$$

For our case we take

$$\mathcal{O} = \Box'^2 - m^4 \tag{33}$$

Introducing the definitions

$$\mathcal{P} = \Box' - m^2, \qquad \mathcal{Q} = \Box' + m^2 \tag{34}$$

we have

$$\mathcal{O} = \mathcal{P}\mathcal{Q}, \qquad Det\,\mathcal{O} = Det\,\mathcal{P}\;Det\,\mathcal{Q} \tag{35}$$

and

$$\mathcal{O}^{-1} = \mathcal{P}^{-1}\,\mathcal{Q}^{-1} = \frac{1}{\Box' - m^2}\,\frac{1}{\Box' + m^2}$$

$$= \frac{1}{2m^2}\left(\frac{1}{\Box' - m^2} - \frac{1}{\Box' + m^2}\right)$$

$$\mathcal{O}^{-1} = \frac{1}{2m^2}\,\mathcal{P}^{-1} - \frac{1}{2m^2}\mathcal{Q}^{-1} \tag{36}$$

So that

$$(32) = \mathcal{N}'(Det\,\mathcal{P})^{-2}(Det\,\mathcal{Q})^{-2} \cdot \exp -i\int \frac{dx}{2m^2}\,\mathcal{T}\,\mathcal{P}^{-1}\bar{\mathcal{T}}\,\exp i\int \frac{dx}{2m^2}\,\mathcal{T}\mathcal{Q}^{-1}\bar{\mathcal{T}} \tag{37}$$

We can use again the gaussian formula (32) for the operators $\mathcal{P}$ and $\mathcal{Q}$ and two independent scalar fields φ_1 and φ_2:

$$\int [\mathcal{D}\varphi_1][\mathcal{D}\bar{\varphi}_1]\,\exp i\int dx\left(\bar{\varphi}_1\mathcal{P}\varphi_1 + \frac{1}{\sqrt{2}m}\,\mathcal{T}\varphi_1 + \frac{1}{\sqrt{2}m}\bar{\mathcal{T}}\bar{\varphi}_1\right)$$
$$= \mathcal{N}_1(Det\,\mathcal{P})^{-2}\,\exp -i\int \frac{dx}{2m^2}\,\mathcal{T}\mathcal{P}^{-1}\bar{\mathcal{T}} \tag{38}$$

$$\int [\mathcal{D}\varphi_2][\mathcal{D}\bar{\varphi}_2]\,\exp i\int dx\left(-\bar{\varphi}_2\mathcal{Q}\varphi_2 + \frac{1}{\sqrt{2}m}\,\mathcal{T}\varphi_2 + \frac{1}{\sqrt{2}m}\,\bar{\mathcal{T}}\bar{\varphi}_2\right)$$
$$= \mathcal{N}_2(Det\,\mathcal{Q})^{-2}\,\exp i\int \frac{dx}{2m^2}\,\mathcal{T}\mathcal{Q}^{-1}\bar{\mathcal{T}} \tag{39}$$

In (38) and (39) we renormalize the scalar fields:

$$\varphi_j \to \sqrt{2}m\varphi_j \qquad (j = 1,2) \tag{40}$$

The equalities (32), (36), (38), and (39) with (33)–(36) and (40) imply that we have established the relation

$$\begin{aligned}
&\int [\mathcal{D}\varphi][\mathcal{D}\bar{\varphi}] \exp i \int dx(\bar{\varphi}\mathcal{P}\mathcal{Q}\varphi + \mathcal{T}\varphi + \mathcal{T}\bar{\varphi}) \\
&\quad = \mathcal{N} \int [\mathcal{D}\varphi_1][\mathcal{D}\bar{\varphi}_1][\mathcal{D}\varphi_2][\mathcal{D}\bar{\varphi}_2] \\
&\qquad \cdot \exp i \int dx\,(2m^2\bar{\varphi}_1\mathcal{P}\varphi_1 - 2m^2\bar{\varphi}_2\mathcal{Q}\varphi_2 + \mathcal{T}(\varphi_1 + \varphi_2) + \mathcal{T}(\bar{\varphi}_1 + \bar{\varphi}_2))
\end{aligned} \tag{41}$$

If we multiply both members of (41) with

$$\exp i \int dx \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \zeta(\partial \cdot A)^2 + \mathcal{K} \cdot A\right) \tag{42}$$

we can deduce, after functional integration over A, that

$$\begin{aligned}
&\int [\mathcal{D}\varphi][\mathcal{D}\bar{\varphi}][\mathcal{D}A] \exp i \int dx\,(\mathcal{L} + \zeta(\partial \cdot A)^2 + \mathcal{T}\varphi + \mathcal{T}\bar{\varphi} + \mathcal{K} \cdot A) \\
&\quad = \mathcal{N} \int [\mathcal{D}\varphi_1][\mathcal{D}\bar{\varphi}_1][\mathcal{D}\varphi_2][\mathcal{D}\bar{\varphi}_2][\mathcal{D}A] \\
&\qquad \cdot \exp i \int dx(\mathcal{L} + \zeta(\partial \cdot A)^2 + \mathcal{T}(\varphi_1 + \varphi_2) + \mathcal{T}(\bar{\varphi}_1 + \bar{\varphi}_2) + \mathcal{K} \cdot A) \\
&\quad = \mathfrak{X}(\mathcal{T}, \mathcal{T}, \mathcal{K})
\end{aligned} \tag{43}$$

where

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + 2m^2\bar{\varphi}_1\mathcal{P}\varphi_1 - 2m^2\bar{\varphi}_2\mathcal{Q}\varphi_2 \tag{44}$$

The equivalence of the generating functionals $\mathfrak{X}(\mathcal{T}, \mathcal{T}, \mathcal{K})$ and $\mathfrak{X}(\mathcal{T}, \mathcal{T}, \mathcal{K})$ expressed by Eq. (43) implies the equivalence of the lagrangians (30) and (44).

The Lagrangian (30) describes the gauge-invariant interaction of the electromagnetic field with a scalar field that obeys a fourth-order equation of motion

$$(\Box'^2 - m^4)\,\varphi = 0 \tag{45}$$

where $\Box'^2$ is given by (11).

On the other hand, the lagrangian (44) describes the interaction of the electromagnetic field with two independent scalar fields obeying second-order equations:

$$(\Box' - m^2)\,\varphi_1 = 0 \tag{46}$$

$$(\Box' + m^2)\,\varphi_2 = 0 \tag{47}$$

with $\Box'$ given by (8).

Equation (46) describes a normal Klein–Gordon particle interacting with the electromagnetic field. Equation (47) describes a tachyonic field interacting with the electromagnetic field. There is not direct interaction between φ_1 and φ_2. The only mutual action is produced via interchange of photons.

The tachyonic mode cannot exist as a free wave. It can only be found virtually, in closed loops, joined to the rest of the diagram by photon lines.

The question of a possible acausal behavior due to its propagator (half-advanced and half-retarded) is answered in ref. 11, and its relation to unitarity is taken up in ref. 12. See also refs. 6 and 13.

6. DISCUSSION

According to previous results [5, 6], tachyons propagate by means of a half-advanced and half-retarded Green-function. This propagator lacks the on-shell δ-function which is present in Feynman's causal function. Consequently, tachyons cannot be found as free particles. Thus, the question of the existence of a possible electric charge for the tachyon seems to be devoid of physical sense. However, a tachyon field can act as an internal carrier of interactions between normal particles. Such is the case, for example, for the higher order equations found in ref. 14.

A field φ obeying

$$(\Box^n - m^{2n})\varphi = 0$$

has n modes of propagation [15]. One of them corresponds to a normal particle of mass m. For n even, there is also a tachyonic mode.

The simplest higher order equation of that family is obtained by taking $n = 2$. It has a bradyon and an associated tachyon mode.

It is natural to introduce the interaction with the electromagnetic field, by using the gauge-covariant derivative. The resulting equation has up to a fourth-order interaction term [Eq. (11)]. The construction of the amplitudes for any electromagnetic process is determined by means of the Feynman diagram technique and the use of vertices (12)–(15) with the propagator (3).

The electromagnetic behavior of a normal Klein–Gordon particle is well known. It corresponds to the use of vertices I_1 Eq. (9), and I_2 Eq. (10), with the propagator K, Eq. (5).

We compare the amplitudes for a given physical process evaluated by means of the second-order theory and those evaluated with the fourth-order theory. Except for a constant factor ($2m^2$) due to normalization [cf. Eq. (6)], the theories give identically equal matrix elements. By algebraic manipulation it is possible to pass from one set of matrix elements to the other. The theories turn out to be completely equivalent in general, as shown in Section 5.

The proof developed in Section 5 sheds light upon the relation between the theories. We have established the equivalence of the Lagrangians $\mathscr{L}$ [Eq. (30)] and $\mathscr{L}$ [Eq. (44)]. The latter Lagrangian corresponds to the theory of two independent second-order equations (46) and (47), while in (45) both modes of excitations live together in the field φ.

We have two equivalent points of view. For the fourth-order Lagrangian (30) photons interact with φ with intensity given by the vertex factors J [Eqs. (12)–(15)]. This interaction excites the propagation of both modes (bradyon and tachyon) according to the Green function G [Eq. (3)]. The construction of the matrix element $M^{(4)}$ is followed by the evaluation of the scattering amplitude and the corresponding cross section.

For the second-order Lagrangian (44) each photon interacts with φ_1 or φ_2 independently, with intensity given by the factors I [Eqs. (9) and (10)]. The excitations produced on φ_1 propagate according to the Green function K. In this way we can construct the matrix element $M^{(2)}$ corresponding to any electromagnetic process. The field φ_2 appears in closed loops which only interact with photons. They influence the polarization tensors of the electromagnetic field (as any charged particle does).

The cross sections are equal to those found with the fourth-order theory. It has been proved in ref. 9 that unitarity holds for tachyon loops, provided that the half-advanced and half-retarded propagator is used. In this reference it is also shown that the propagation associated with a Wheeler Green function takes place inside the light cone of the coordinates. It is never superluminal (not even for tachyons).

Summarizing: The electromagnetic behavior of a charged Klein–Gordon particle and that of the bradyon component of a field φ obeying the fourth-order equation (45) are equal. The tachyon component only acts in internal loops coupled to photon lines.

ACKNOWLEDGMENTS

This work was partially supported by Consejo Nacional de Investigaciones Cientificas and Comision de Investigaciones Cientificas de la Pcia. de Buenos Aires, Argentina.

REFERENCES

[1] G. Feinberg, *Phys. Rev.* **159**, 1089 (1957).
[2] E. Recami, *Nuovo Cimento*, **N.6** (1986).
[3] E. C. G. Sudarshan, In *Elementary Particle Theory Nobel Symposium*, **Vol. 8**, N. Svartholm, ed., Stockholm (1968), p. 335.
[4] G. Ecker, *Ann. Phys.* (NY) **58**, 303 (1970).
[5] D. G. Barci, C. G. Bollini, and M. C. Rocca, *Nuovo Cimento* **106A**, 603 (1993).
[6] D. G. Barci, C. G. Bollini, and M. C. Rocca, *Int. J. Mod. Phys. A* **9**, 3497 (1994).
[7] J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys. A* **17**, 157 (1945).
[8] D. G. Barci, C. G. Bollini, and M. C. Rocca, *Int. J. Mod. Phys. A* **10**, 1737 (1995).
[9] H. Schnitzer and E. C. G. Sudarshan, *Phys. Rev.* **123**, 219 (1969).
[10] L. D. Faddeev and A. A. Slavnov, *Gauge Fields. Introduction to Quantum Theory*, Benjamin-Cummings (1970).
[11] J. A. Wheeler and R. P. Feynman, *Rev. Mod. Phys. A* **21**, 425 (1949).
[12] C. G. Bollini and M. C. Rocca, The Wheeler propagator, *Int. J. Theor. Phys.*, to appear.
[13] C. G. Bollini and L. E. Oxman, *Int. J. Mod. Phys. A* **8**, 3185 (1993).
[14] C. G. Bollini and J. J. Giambiagi, *Phys. Rev. D* **32**, 3316 (1985).
[15] D. G. Barci, C. G. Bollini, and M. C. Rocca, *Int. J. Mod. Phys. A* **9**, 4169 (1994).